A MEASURE FOR THE STABILITY OF SOLID STATE NOISE SOURCES

Motohisa Kanda
Electromagnetics Division
National Bureau of Standards
Boulder, Colorado 80302

Abstract

A measure for the stability of solid state noise sources is discussed. Its applicability is demonstrated. A technique similar to cross correlation is employed to separate the instabilities of the noise source from those of the measurement system.

I. Introduction

Solid state noise sources are influenced by their environment, and an unsuitable environment can cause unstable operation. Observed instabilities can be traced to changes in ambient temperature, operating power supply voltages and currents, humidity, etc. In order to unravel the causes of these instabilities, the measurements of the stability of solid state noise sources have received considerable attention.

The purpose of this paper is to describe a measure of stability applicable to solid state noise sources. The stability of the output noise from solid state noise sources is compared with that from argon gas discharge noise sources at several frequencies between 1 and 18 GHz. To separate the instability of the noise sources from that of the measurement system, a technique similar to cross correlation is employed.

This measure of stability allows one to specify the performance of solid state noise sources and facilitates meaningful relative comparisons among these devices.

2. Allan Variance Analysis

An important performance parameter for a solid state noise source is the stability of its mean square output noise as a function of sampling time interval. For some models of noise generation mechanisms which seem to be representative of solid state noise sources, the classical variance is unbounded. A well-behaved and convergent stability measure is, therefore needed. Such has been developed for the field of frequency stability, and the recommended measure for the time-domain frequency stability is an Allan variance.[1,2,3] A special case of an Allan variance analysis[1] is used in this paper to establish a measure of stability for solid state noise sources.

A record of the phenomenon under consideration, $y(t)$, is divided into a number of equal time segments of length τ, and the average value of $y(t)$ of each segment, y_k, is calculated by

$$y_k = \frac{1}{\tau} \int_{t_k}^{t_k+\tau} y(t)\, dt, \qquad (1)$$

where y_k is the k^{th} segment average starting at time t_k. Next, the sample variance (sample size two), $\sigma_y^2(2,\tau)$, of successive averages is calculated. That is

$$\sigma_y^2(2,\tau) = \sum_{n=k}^{k+1} (y_n - \bar{y}_k)^2 = \frac{1}{2}(y_{k+1} - y_k)^2, \qquad (2)$$

where

$$\bar{y}_k \equiv \frac{1}{2} \sum_{n=k}^{k+1} y_n \qquad (3)$$

is the average of the two successive segment averages y_k and y_{k+1}. The Allan variance, $\sigma_y^2(\tau)$, for this special case (sample size two) is then defined to be[1]

$$\sigma_y^2(\tau) \equiv \langle \sigma_y^2(2,\tau) \rangle, \qquad (4)$$

where the brackets represent the infinite time average of $\sigma_y^2(2,\tau)$ over all pairs of successive y_k constructed from $y(t)$. In practice, a finite set of $\sigma_y^2(2,\tau)$ often gives a good estimate of $\sigma_y^2(\tau)$.

The preceding calculation is repeated for various values of sampling time interval, τ. For a given maximum allowable deviation in $y(t)$ the maximum sampling time interval can then be determined. When the noise generation mechanism is random and un-autocorrelated, the Allan variance, $\sigma^2(\tau)$, decreases as the inverse of sampling time interval, τ. The data analysis is typically performed by a computer via a program designed to compute the appropriate Allan variance. In the computer program logarithm σ versus logarithm τ is plotted. An example is given below.

3. Measurement Results

A technique similar to cross correlation is employed in order to separate the instability due to the noise source from that due to the measurement system. Two independent noise sources, a solid state noise source and an argon gas discharge noise source, are used for this purpose.

When X_{sm} and Y_{gm} are, respectively, the time series of observables for the output noises from a solid state noise source and from an argon gas discharge noise source, the cross correlation with zero delay time, $R_{X,Y}(0)$ is defined to be

$$R_{X,Y}(0) \equiv \frac{\langle X_{sm} \cdot Y_{gm} \rangle}{\sqrt{\langle X_{sm}^2 \rangle \langle Y_{gm}^2 \rangle}}, \quad (5)$$

where the brackets represent the infinite time average. Then, one can show that the Allan variance for the measurement system $\sigma_m^2(\tau)$ is given by

$$\sigma_m^2(\tau) = R_{\sigma_X,\sigma_Y}(0) \cdot \sigma_X(\tau) \cdot \sigma_Y(\tau) \quad (6)$$

and

$$\sigma_X^2(\tau) = \sigma_s^2(\tau) + \sigma_m^2(\tau) \quad (7)$$

$$\sigma_Y^2(\tau) = \sigma_g^2(\tau) + \sigma_m^2(\tau) \quad (8)$$

where $\sigma_s^2(\tau)$, $\sigma_g^2(\tau)$, and $\sigma_m^2(\tau)$ are, respectively, Allan variances for a solid state noise source, an argon gas discharge noise source, and the measurement system. Here it is assumed that the solid state noise source, the argon gas discharge noise source, and the measurement system are independent of and un-crosscorrelated with each other. Thus, by use of equation (6) on appropriate data, it is possible to extract the instability due to a solid state noise source from that due to the measurement system.

We have measured $\sigma_X^2(\tau)$ and $\sigma_Y^2(\tau)$ as a pair of time series. However, we did not make measurements of X and Y strictly simultaneously. Rather, measurements of X were alternated in time with measurements of Y. Thus, the variance $\sigma_X^2(\tau)$ (and $\sigma_Y^2(\tau)$) measured in this manner contains distributed dead time in itself. For the purpose of this paper, we formally assume that the values of this variance are equal to the values of corresponding Allan variance with no dead time.

We are able to approximately estimate $\sigma_m^2(\tau)$ by computation of $R_{\sigma_X,\sigma_Y}(0)$ from our measured data and by use of it in equation (6). Because the instability of the measurement system does not have a white spectrum, but instead is divergent toward lower frequencies, equation (6) and this procedure are approximately valid. The analysis for the limits of the validity of this use of equation (6) will be prepared and is the subject of a future paper.[4]

Figure 1 shows the results of the Allan variance analysis for a typical solid state noise source. The square root of the Allan variance of $\delta T/T$, where T is the output noise from the solid state noise source is 0.0014 ($10 \log_{10}(1 + \sigma_{\delta T/T}) = 0.006$ dB) for 1-day sampling time interval. Since the Allan variance increases with an increase of sampling time interval, the noise generation mechanism for the solid state noise source is correlated with itself. To compare the stability of the solid state noise source with that of a typical argon gas discharge noise source, the square root of the Allan variance of a typical argon gas discharge noise source is also shown in Figure 1. The square root of the Allan variance of $\delta T/T$, where T is the output noise from the argon gas discharge noise source is about 0.0002 ($10 \log_{10}(1 + \sigma_{\delta T/T}) = 0.001$ dB) for 1-day sampling time interval, and the stability of the argon gas discharge noise source is found to be very good.

Although it is beyond the scope of this paper to describe the causes of instability in the solid state noise sources in detail, the effects due to two main environmental influences, ambient temperature and a supply current, are examined below.

The temperature dependence of the solid state noise source is shown in Figure 2. It is obvious from the data that there is no measurable ambient temperature dependence of the output noise as long as a constant current power is used to operate the solid state noise source.

The output noise from the solid state noise source as a function of the dc operating current is plotted in Figure 3 using logarithmic scales. The solid line shows I^{-1} behavior, where I is the bias current, as predicted from the theory of Hines.[5] These results suggest that the output noise approaches the predicted I^{-1} behavior except at extremely low bias currents, where the avalanche discharge in the solid state noise source becomes unstable and nonuniform, and the measurements lose their accuracy. Figure 4 shows experimentally the dependence on the operating current for the output noise from several argon gas discharge noise sources. It is empirically found that the output noise from the argon gas discharge noise source follows $I^{-0.1}$ behavior where I is its dc discharge current.

The stable operation of a solid state noise source is more strongly dependent on its operating current than is an argon gas discharge noise source. Therefore, a stable operating current should be used in the operation of solid state noise sources.

4. Summary and Conclusions

A concise determination for the measurement of stability has been given using an Allan variance analysis. The measure was applied to the evaluation of stability in solid state noise sources. It is found that the square root of the variance for the output noise from a typical solid state noise source for 1-day sampling time interval is typically 0.0014, or 0.006 dB. In contrast, the square root of the variance for the output noise from a typical argon gas discharge noise source for 1-day sampling time interval is about 0.0002, or 0.001 dB. A technique similar to cross correlation was employed to separate the stability of the noise sources under consideration from that of the measurement system. A brief study indicates that the operating current of a solid state noise source is one of the important environmental factors which influence the stable operation of the solid state noise source.

It is hoped that this measure of stability obtained using the Allan variance analysis approach allows one to specify the performance of solid state noise sources under consideration and facilitates more meaningful relative comparisons among these devices.

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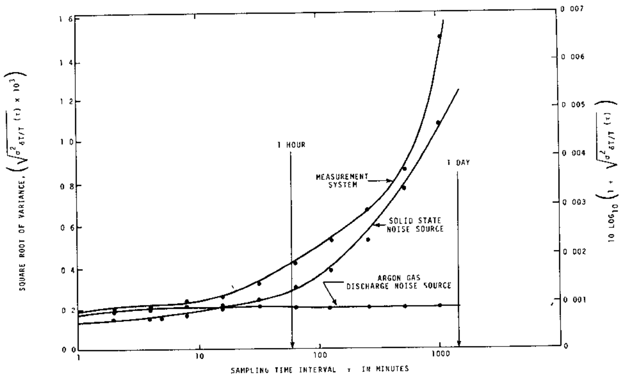

Figure 1. Allan Variance of Fluctuations of Mean Square Intensity of Output Noise as a Function of Sampling Time Interval, τ. (No Confidence Limits are Known. See Text)

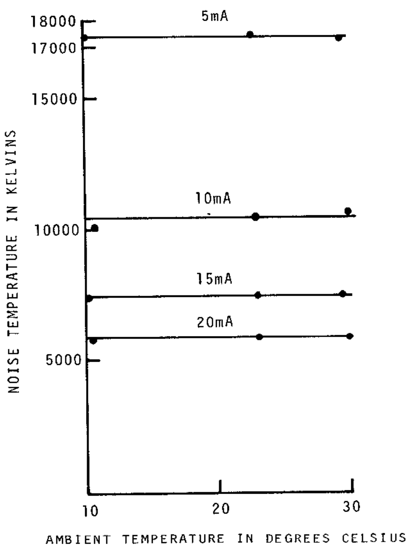

Figure 2. Output Noise from a Solid State Noise Source as a Function of Ambient Temperature.

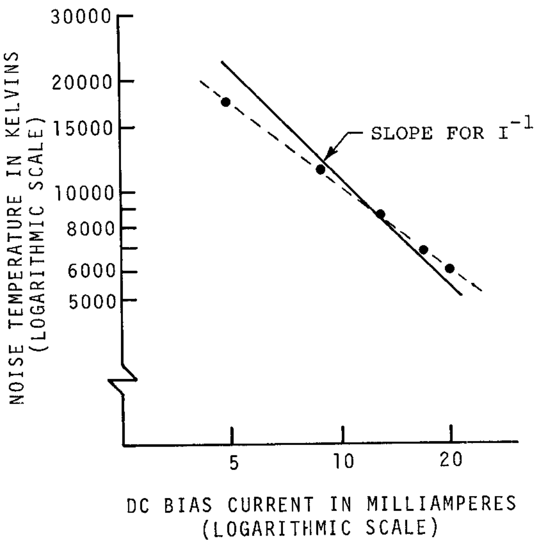

Figure 3. Output Noise from a Solid State Noise Source as a Function of DC Bias Current.

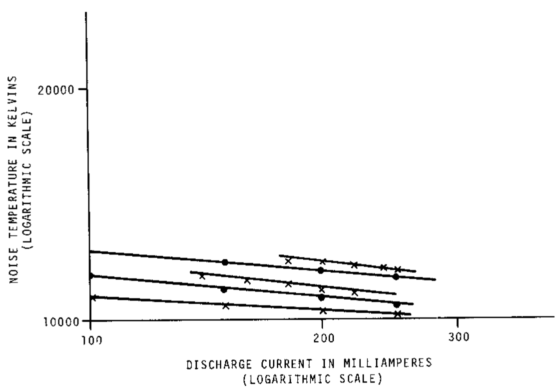

Figure 4. Output Noise from Five Different Argon Gas Discharge Noise Sources as a Function of DC Discharge Current.